Supported by tetfund TERTIARY EDUCATION TRUST FUND

VISCOUS AND JOULE DISSIPATION EFFECTS ON HEAT TRANSFER IN MHD POISEUILLE FLOW IN THE PRESENCE OF RADIAL MAGNETIC FIELD

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Received: January 03, 2016 **Accepted:** March 13, 2017

Abstract: This study investigates analytically the effects of pressure forces, radial magnetic field, viscous and Joule dissipations on the Nusselt number for a steady, fully developed MHD pressure driven flow through an asymmetrically heated annulus of two infinitely concentric cylinders. The surfaces of both cylinders are kept at unequal temperatures. The governing momentum and energy equations are transformed by defining relevant dimensionless variables and solved analytically using the method of undetermined coefficient. The effects of various controlling parameters on the flow are graphically presented. Significant result from the present work is that, increase in Hartmann number has a marked effect on the temperature distribution for cases of Brinkman number ($Br \neq 0$) which is a case of heat generation due to viscous and Joule dissipation, while in the absence of dissipation, the Hartmann number has an insignificant effect on the temperature profile. In addition, the Nusselt number at the inner surface of the outer cylinder displays an unbounded swing which is significantly influenced by the degree of asymmetric heating.

Keywords: Poiseuille flow, MHD, viscous dissipation, Joule dissipation, Nusselt numbers

Introduction

An increasing interest of study have been drawn over the years to fluid flow through annular regions and the resulting heat transfer have assumed considerable industrial significance in the light of advancement in hydraulics and nuclear technology. In such industrial applications, the study of heat transfer in pressure driven flows is of great importance. Thus, the influence of viscous dissipation is very crucial in heat transfer analysis associated with the applied pressure. The earliest theoretical work concerning the heat generation due to viscous dissipation was carried out by Brinkman (1951), Some available literature in the area of convective heat transfer revealed that temperature distribution in the fluid is uniform when both boundaries are kept at equal temperatures (symmetric heating) while at unequal wall temperatures (asymmetric heating), the temperature distribution is a linear function of the transverse coordinate (Cheng *et al.*, 1990; Hamadah and Wirtz, 1991; Aung and Worku, 1986).

In the case of unequal wall temperatures, heat transfer between the two boundaries occur by pure conduction (conductionregime) and the absence of viscous dissipation makes the temperature profile to be independent irrespective of the asymmetry of the wall heating. Moreover, the velocity profile is affected by the buoyancy forces and can give rise to reversal flow both for downward and upward flows. Recently, Ramjee and Satyamurty (2010) investigated the Nusselt number for viscous dissipation flow between parallel plates kept at unequal wall temperatures. Jambal *et al*. (2005) studied the effects of viscous dissipation and fluid axial heat conduction on heat transfer for non- Newtonian fluids in ducts with uniform wall temperature. Lahjomri *et al*. (2002) examined analytically, the thermally developing Hartmann flow through a parallel plate channel including viscous dissipation, Joule heating and axial heat conduction with a step change in wall temperatures. Aydin and Avci (2006) investigated viscous dissipation effects on the heat transfer in a Poiseuille flow. Kumar and Satyamurty (2011) studied the limiting nusselt number for laminar forced convection in asymmetrically heated annuli with viscous dissipation. Mondal and Mukherjee (2012) examined the effects of viscous dissipation on the limiting value of Nusselt number for a shear driven flow through an asymmetrically heated annulus.

In the last few decades, the study of electrically conducting fluid flows through different channels in the presence of magnetic field (MHD) has attracted the interest of many researchers due to its applications in engineering and sciences. It is applied industrially in polymer technology, MHD power generators, MHD pumps and purification of metals. One of the first works on MHD fluid flow through an annulus was discussed by Globe (1959), where the fully developed laminar MHD flow in an annular channel is presented. Natural convection flow in vertical annuli under radial magnetic field has been investigated by Singh *et al.* (1997).

Antimirov and Kolyshkin (1984) investigated the unsteady MHD flow in an annular channel with radial magnetic field. Emad and Mohamed (2005) presented the effects of viscous dissipation and Joule heating on MHD free convection flow through a semi-infinite vertical plate.In another related work, Mozayyeni and Rahimi (2012) numerically investigated mixed convection flow in a fully developed region between two concentric cylinders of infinite length in the presence of constant radial magnetic field. Heat transfer in an unsteady MHD flow of a non-Gray optically thin fluid through an infinite concentric cylinders with radiation is examined by Nazibuddin and Manas (2015). Recently, Chamkha (2000) studied the unsteady laminar MHD flow and heat transfer in channels and circular pipes taking into account the effects of oscillating and ramped pressure gradients

The objective of this article is to investigate analytically the effects of radial magnetic field including viscous and Joule dissipation on the steady fully developed Poiseuille flow of a viscous incompressible, electrically conducting fluid through an asymmetrically heated annulus. The present work is further extension of Mondal and Mukherjee (2012) as a pressure induced flow in the presence of radial magnetic field.

Mathematical and Methods

In this section, the flow geometry and its corresponding mathematical models are solved and solutions of the Nusselt numbers at both walls of the annulus are presented. The flow is considered to be a steady fully developed flow of a viscous, incompressible and electrically conducting fluid through an asymmetrically heated annulus of two stationary concentric cylinders of infinite length with inner radius R_1 and outer radius R_2 as shown in Fig. 1. The flow is induced by pressure applied in the axial direction. No-slip boundary conditions are assumed to be valid at both plates for both hydro-dynamically and thermally developed flows. For the thermal boundary conditions, the inner and outer surfaces of the annulus are kept at unequal constant temperatures T_1 and T_2 respectively. The z-axis is taken along the axis of the cylinder and r-axis is in the radial direction from the centre of the cylinder. Moreover, $\bar{T}$ is the average surface temperature and T_f the fluid temperature. The unequal temperature at both surfaces is accounted for by a parameter (β) to characterize the asymmetry in surface heating. Furthermore, some important assumptions for the present study are:

- ✓ No heat generation and constant thermo-physical properties;
- ✓ Negligible axial conduction in the fluid through the wall;
- ✓ Fully developed Incompressible flow;
- ✓ Small magnetic Reynolds number.

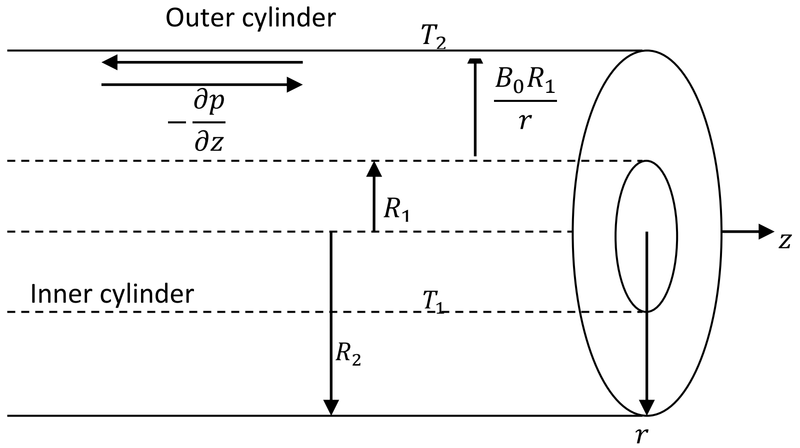

Fig. 1: Physical geometry of the problem

A magnetic field of the form B_0R_1/r, is directed radially outward and it corresponds to negligibly induced magnetic field compared to the externally applied one. The governing equations of momentum and energy in dimensional form neglecting convective terms are given as follows:

$$v\frac{1}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)-\frac{\sigma B_0^2 u}{\rho r^2}-\frac{1}{\rho}\frac{\partial p}{\partial z}=0 \qquad (1)$$

$$k\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\mu\left(\frac{du}{dr}\right)^2+\frac{\sigma B_0^2 u^2}{\rho r^2}=0 \qquad (2)$$

Subject to the relevant boundary conditions:

$$u=0 \; and \;\; T=T_1 \; at \; r=R_1$$
$$u=0 \; and \; T=T_2 \; at \; r=R_2 (3)$$

Defining the following dimensionless quantities:

$$V=\frac{u}{U_0}, R^*=\frac{R_1}{R_2}, \theta=\frac{(T-\bar{T})}{(T_f-\bar{T})}, Br=\frac{\mu {U_0}^2}{k(T_f-\bar{T})}, M^2=\frac{\sigma B_0^2 (R_2-R_1)^2}{\rho v},$$
$$R=\frac{(r-R_1)}{(R_2-R_1)}, P=\frac{(R_2-R_1)^2}{U_0 v}\frac{1}{\rho}\left(-\frac{\partial P}{\partial z}\right), \bar{T}=\frac{(T_2+T_1)}{2} (4)$$

Using equation (4) above, the dimensionless form of momentum and energy equations are:

$$\frac{1}{[R^*+(1-R^*)R]}\frac{d}{dR}\left[[R^*+(1-R^*)R]\frac{dV}{dR}\right]-\frac{M^2V}{[R^*+(1-R^*)R]^2}+P=0 \qquad (5)$$

$$\frac{1}{[R^*+(1-R^*)R]}\frac{d}{dR}\left[[R^*+(1-R^*)R]\frac{d\theta}{dR}\right]+Br\left[\left(\frac{dV}{dR}\right)^2+\frac{M^2V^2}{[R^*+(1-R^*)R]^2}\right]=0 \qquad (6)$$

Subject to the boundary conditions

$$V=0, \theta=-\frac{(1-\beta)}{(1+\beta)} \; at \;\; R=0$$
$$V=0, \theta=\frac{(1-\beta)}{(1+\beta)} \; at \; R=1 \qquad (7)$$

Where$\beta=\frac{(T_2-T_f)}{(T_1-T_f)}$

Applying the method of undetermined coefficient, equation (5) yields

$$V(R)=P\{a_1[R^*+(1-R^*)R]^{\lambda}+a_2[R^*+(1-R^*)R]^{-\lambda}-\; a_3[R^*+(1-R^*)R]^2\} \qquad (8)$$

The pressure gradient P is calculated from the expression of mean velocity given by:

$$U_0 = \frac{2\int_{R_1}^{R_2} ru(r)dr}{(R_2^2 - R_1^2)} \quad (9)$$

Substituting equation (8) into the energy equation (6) and solving it using boundary conditions (7) gives the dimensionless temperature profile as:

$$\theta(R) = a_{11}\log\{[R^* + (1-R^*)R]\} + a_{12} - BrP^2\{a_4[R^* + (1-R^*)R]^{2\lambda} + a_5[R^* + (1-R^*)R]^{-2\lambda} + a_6[R^* + (1-R^*)R]^4 + a_7[R^* + (1-R^*)R]^{\lambda+2} + a_8[R^* + (1-R^*)R]^{-\lambda+2}\} \quad (10)$$

The dimensionless bulk temperature θ_b is defined as:

$$\theta_b = \frac{\int_0^1 [R^* + (1-R^*)R]V\theta dR}{\int_0^1 [R^* + (1-R^*)R]VdR} \quad (11)$$

Solving equation (11) we obtain:

$$\theta_b = \frac{4a_{36}(\lambda^2 - 4)}{a_{14}} \quad (12)$$

Nusselt numbers based on bulk temperature as the reference fluid temperature are expressed as

$$Nu_1 = -\frac{1}{[\theta(0) - \theta_b]}\frac{d\theta}{dR}\bigg|_{R=0} \quad (13)$$

$$Nu_2 = \frac{1}{[\theta(1) - \theta_b]}\frac{d\theta}{dR}\bigg|_{R=1} \quad (14)$$

Whereθ_1 *and* θ_2 are the dimensionless temperatures at the inner and outer surfaces of the annulus. Using equation (13) *and* (14), Nusselt number at the outer surface of the inner cylinder and inner surface of the outer cylinder are given respectively as

$$Nu_{1c} = \frac{[(1-R^*)/R^*][(a_{13}a_{37}R^*) + a_{11}]}{\frac{(1-\beta)}{(1+\beta)} + \frac{4a_{36}(\lambda^2-4)}{a_{14}}} \quad (15)$$

$$Nu_{2c} = \frac{(1-R^*)(a_{13}a_{38} + a_{11})}{\frac{(1-\beta)}{(1+\beta)} - \frac{4a_{36}(\lambda^2-4)}{a_{14}}} \quad (16)$$

Constants used in the present work are:

$$\lambda = \frac{M}{(1-R^*)}, P = \frac{[1-(R^*)^2]4(\lambda^2-4)}{2a_{14}}, a_1 = \frac{[(R^*)^{-\lambda} - (R^*)^2]}{(1-R^*)^2(4-\lambda^2)[(R^*)^{-\lambda} - (R^*)^{\lambda}]},$$

$$a_2 = \frac{[(R^*)^2 - (R^*)^{\lambda}]}{(1-R^*)^2(4-k^2)[(R^*)^{-\lambda} - (R^*)^{\lambda}]}, a_3 = \frac{1}{(1-R^*)^2(4-\lambda^2)}, a_4 = \frac{a_1^2}{2}, a_5 = \frac{a_2^2}{2},$$

$$a_6 = \frac{a_3^2(4+\lambda^2)}{16}, a_7 = -\frac{2a_1a_3\lambda(2+\lambda)}{(\lambda+2)^2}, a_8 = \frac{2a_2a_3\lambda(2-\lambda)}{(\lambda-2)^2}, a_9 = a_4 + a_5 + a_6 + a_7 + a_8$$

$$a_{10} = [a_4(R^*)^{2\lambda} + a_5(R^*)^{-2\lambda} + a_6(R^*)^4 + a_7(R^*)^{\lambda+2} - a_8(R^*)^{-\lambda+2} - a_9],$$

$$a_{11} = \frac{BrP^2}{\log(R^*)} + 2\frac{(1-\beta)}{(1+\beta)}, a_{12} = \frac{(1-\beta)}{(1+\beta)} + a_9BrP^2, a_{13} = -BrP^2$$

$$a_{14} = [4a_1(\lambda-2)(1-(R^*)^{\lambda+2}) + 4a_2(\lambda+2)((R^*)^{-\lambda+2} - 1) + a_3(\lambda^2-4)((R^*)^4 - 1)]$$

$$a_{15} = \frac{a_1a_4a_{13}(1-(R^*)^{3\lambda+2})}{3\lambda+2}; a_{16} = \frac{a_1a_5a_{13}((R^*)^{-\lambda+2} - 1)}{\lambda-2}; a_{17} = \frac{a_1a_6a_{13}(1-(R^*)^{\lambda+6})}{\lambda+6}$$

$$a_{18} = \frac{a_1a_7a_{13}(1-(R^*)^{2\lambda+4})}{2\lambda+4}; a_{19} = \frac{a_1a_8a_{13}(1-(R^*)^4)}{4}$$

$$a_{20} = a_1a_{11}\left[\frac{(R^*)^{\lambda+2}}{(\lambda+2)^2} - \frac{(R^*)^{\lambda+2}\log(R^*)}{\lambda+2} - \frac{1}{(\lambda+2)^2}\right]; a_{21} = \frac{a_1a_{12}(1-(R^*)^{\lambda+2})}{\lambda+2}$$

$$a_{22} = \frac{a_2a_4a_{13}(1-(R^*)^{\lambda+2})}{\lambda+2}; a_{23} = \frac{a_2a_5a_{13}((R^*)^{-3\lambda+2} - 1)}{3\lambda-2}; a_{24} = \frac{a_2a_6a_{13}((R^*)^{-\lambda+6} - 1)}{\lambda-6}$$

$$a_{25} = \frac{a_2a_7a_{13}(1-(R^*)^4)}{4}; a_{26} = \frac{a_2a_8a_{13}((R^*)^{-2\lambda+4} - 1)}{2\lambda-4};$$

$$a_{27} = a_2a_{11}\left[\frac{(R^*)^{-\lambda+2}}{(\lambda-2)^2} + \frac{(R^*)^{-\lambda+2}\log(R^*)}{\lambda-2} - \frac{1}{(\lambda-2)^2}\right]; a_{28} = \frac{a_2a_{12}((R^*)^{-\lambda+2} - 1)}{\lambda-2}$$

$$a_{29} = \frac{a_3a_4a_{13}((R^*)^{2\lambda+4} - 1)}{2\lambda+4}; a_{30} = \frac{a_3a_5a_{13}(1-(R^*)^{-2\lambda+4})}{2\lambda-4}; a_{31} = \frac{a_3a_6a_{13}((R^*)^8 - 1)}{8}$$

$$a_{32} = \frac{a_3a_7a_{13}((R^*)^{\lambda+6} - 1)}{\lambda+6}; a_{33} = \frac{a_3a_8a_{13}(1-(R^*)^{-\lambda+6})}{\lambda-6}$$

$$a_{34} = a_3a_{11}\left[\frac{1}{16} + \frac{(R^*)^4\log(R^*)}{4} - \frac{(R^*)^4}{16}\right]; a_{35} = \frac{a_3a_{12}((R^*)^4 - 1)}{4}$$

$$a_{36} = (a_{15} + a_{16} + a_{17} + a_{18} + a_{19} + a_{20} + a_{21} + a_{22} + a_{23} + a_{24} + a_{25} + a_{26} + a_{27} + a_{28} + a_{29} + a_{30} + a_{31} + a_{32} + a_{33} + a_{34} + a_{35})$$

$$a_{37} = [(2\lambda a_4(R^*)^{2\lambda-1}) - (2k\lambda(R^*)^{-2\lambda-1}) + (4a_6(R^*)^3) + (a_7(\lambda+2)(R^*)^{\lambda+1}) + (a_8(-\lambda+2)(R^*)^{-\lambda+1})]$$

$$a_{38} = \left[(2\lambda a_4) - (2\lambda a_5) + 4a_6 + \left(a_7(\lambda + 2)\right) + \left(a_8(-\lambda + 2)\right)\right]$$

Results and Discussions

This study investigates the effects of varying pressure gradient, viscous and Joule dissipations on the velocity and temperature profiles as well as Nusselt numbers on both surfaces of the annulus in the presence of radial magnetic field. The influence of some controlling parameters (pressure gradient, Hartmann number, Brinkman number and degree of asymmetric heating) is presented graphically in Figs. $2 - 7$. The Hartmann and Brinkman numbers are taken over the intervals $2.0 \leq M \leq 3.0$ and $-0.5 \leq Br \leq 0.5$ respectively while radius ratio $R^* = 0.5$. Also, three different degrees of asymmetric parameter $\beta = -0.5, 0.5\ and\ 1$ have been considered. $Br > 0$ correspond to the case where the surfaces are been heated. In addition, the curves for Brinkman number $(Br \neq 0)$ and Hartmann number $(M \neq 0)$, represents heat generation by viscous and Joule disspation. Fig. 2 shows the effect of pressure forces and Hartmann number (M) on the velocity profile. As Hartmann number (M) increases, fluid velocity decreases near the inner cylinder while a flow reversal is observed near the outer cylinder. This trend is attributed to the enhancement in the bouyancy effect due to increase dissipation. Energy generated by dissipation increases the bouyancy or pressure forces which in turn enhances the velocity of the fluid.

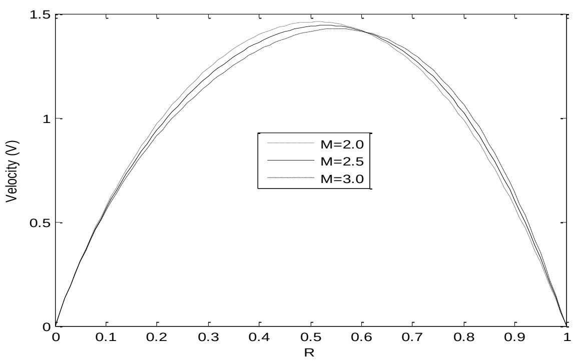

Fig. 2: Velocity profile for different values of Hartmann number (M)

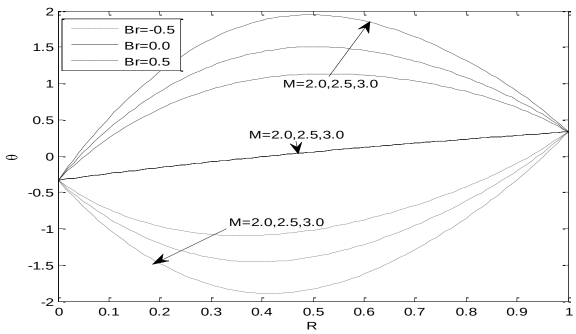

Fig. 3(a): Variation of temperature for different values of Hartmann number (M) for $\beta = 0.5$

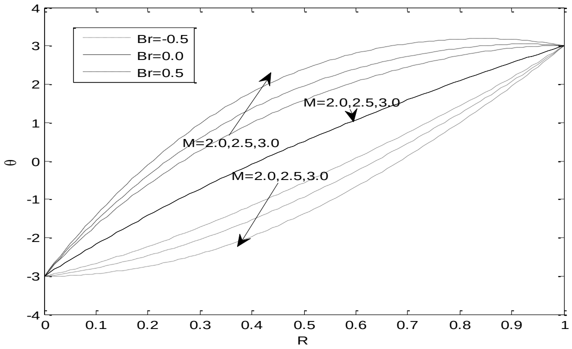

Fig. 3(b): Variation of temperature for different values of Hartmann number (M) for $\beta = -0.5$

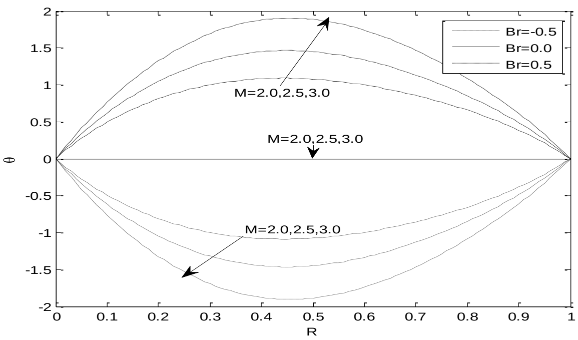

Fig. 3(c): Variation of temperature for different values of Hartmann number (M) for $\beta = 1.0$

In Fig. $3(a)\ to\ (c)$, the variation of temperature for different values of Hartmann number (M) and Brinkman number (Br), obtained at three different degree of asymmetric surface heating ($\beta = 0.5, -0.5\ and\ 1$) respectively are presented. It is be observed from these figures as well as equation (6) that, increase in Hartmann number (M) has a marked effect on the temperature distribution for cases of Brinkman number $(Br \neq 0)$ which is a case of heat generation due to viscous and Joule dissipation, while in the absence of dissipation, the Hartmann number (M) has an insignificant effect on the temperature profile. In fact, for positive value of Brinkman number (Br) and increase in Hartmann number (M), heat generation inside the fluid increases due to viscous and Joule dissipations, and this results to enhancement in fluid temperature. Hence, the presence of dissipation tends to distort the temperature profile in comparison with the situation when dissipation is absent. Fig. $4(a) and\ (b)$ depicts the variation of bulk temperature with radius ratio R^* for different values of Hartmann number (M) corresponding to two cases of asymmetric surface heating ($\beta = 0.5\ and -0.5$), respectively. It is observed from both figures for different values of Brinkman number (Br), that increase in Hartmann number (M) leads to increase in bulk temperature for positive value of Brinkman number (Br) while the reverse effect is observed for negative values of Brinkman number (Br). It is important to note from both figures that, the bulk temperature decreases as the radius ratio R^* increases.

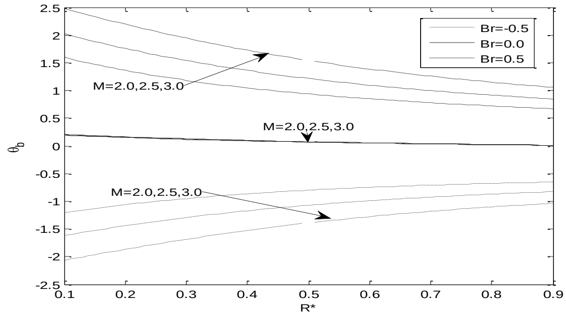

Fig. 4(a): Bulk temperature profile for different values of M for $\beta = 0.5$

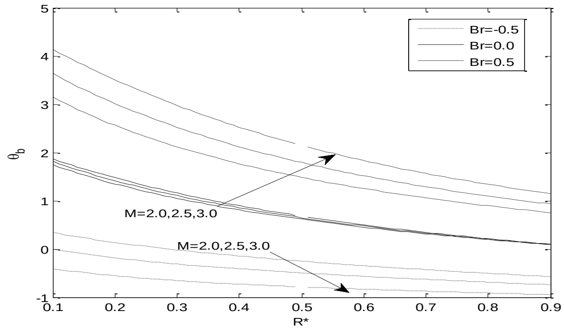

Fig. 4(b): Bulk temperature profile for different values of M for $\beta = -0.5$

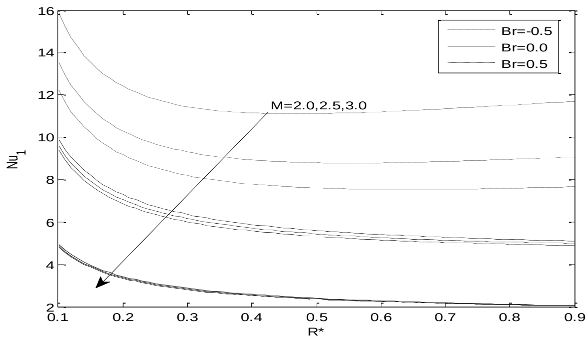

Fig. 5(a): Variation of Nu_1 with R^* for different values of M for $\beta = 0.5$

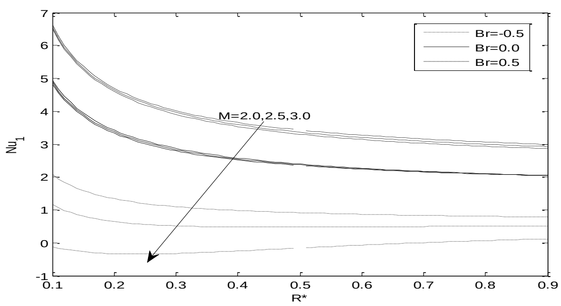

Fig. 5(b): Variation of Nu_1 with R^* for different values of M for $\beta = -0.5$

Fig. $5(a) and\ (b)$, illustrates the effect of Hartmann number (M) on the Nusselt number at the outer surface of the inner cylinder for two cases of asymmetric surface heating and for different values of Brinkman number (Br). It is evident from both figures that increase in Hartmann number (M) leads to decrease in Nusselt number on the outer surface of the inner cylinder. This effect is more visible at the entrance region of the inner cylinder for Brinkman number $(Br = -0.5)$. It is also oberved that, the Nusselt number at the outer surface of the inner cylinder decreases as the radius ratio R^* increases. Fig. $6(a) and\ (b)$ depicts the variation of Nusselt number on the inner surface of the outer cylinder for different values of Hartmann number (M). It is noticed from both figures that the Nusselt number on the inner surface of the outer cylinder decreases as Hartmann number (M) increases for all cases of asymmetric surface heating. It is observed from Fig. $6(a)$, that for Brinkman number $(Br = 0.5)$, Hartmann number (M) has lesser effect at the entrance region as compared with its effect at the exit region of the outer cylinder, while from Fig. $6(b)$, the Nusselt number at the inner surface of the outer cylinder displays an unbounded swing for Brinkman number $(Br = 0.5)$.

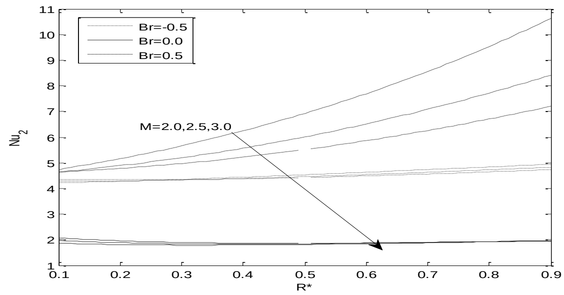

Fig. 6(a) Variation of Nu_2 with R^* for different values of M for $\beta = 0.5$

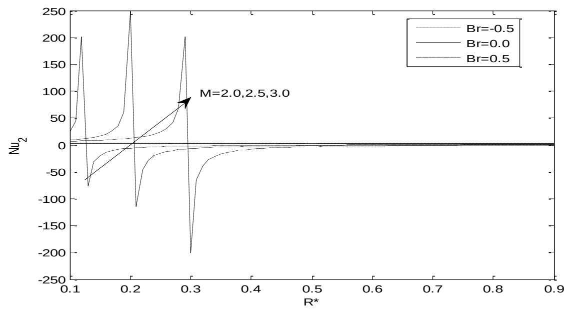

Fig. 6(b): Variation of Nu_2 with R^* for different values of M for $\beta = -0.5$

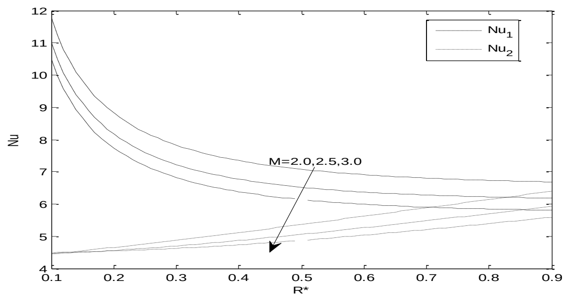

Fig. 7: Variation of Nu_c for different values of M for $\beta = 1$.

The variation of Nusselt number on both surfaces (outer surface of the inner cylinder and inner surface of the outer cylinder) for different values of Hartmann number (M) for the case of symmetric surface heating $(\beta = 1)$ is presented in Fig. 7. It is cruical to note that at equal temperature, the Brinkman number (Br) has an insignificant effect on the Nusselt numbers at both surfaces, and this shows similarity with the result obtained by Mondal and Mukherjee (2012) for a shear driven flow. Moreover, increase in Hartmann number (M) leads to contraction in Nusselt number on both surfaces of the annulus. It is evident that the Hartmann number (M)

has a more pronounced effect on the outer surface of the inner cylinder as compared with its effect on the inner surface of the outer cylinder when both surfaces are kept at equal temperatures.

Conclusion

This study investigates the steady fully developed Poiuseille flow in a horizontal annulus in the presence of radial magnetic field with viscous and Joule dissipation. The effects of some controlling parameters on the flow are discussed. Significant deductions made from the present study are as follows: Increase in Hartmann number (M) and radius ratio R^* leads to increase and decrease in pressure gradient respectively; Flow reversal is observed on the velocity profile due to buoyancy effects; The Nusselt number on the inner surface of the outer cylinder displays an unbounded swing when viscous and Joule dissipations are considered, and At equal surface temperature, Brinkman number (Br) has an insignificant effect on the Nusselt numbers at both surfaces.

Conflict of Interest

The authors declare that there is no conflict of interest regard whatsoever.

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Nomencleture

Br	Brinkman number
k	thermal conductivity
M	Hartmann number
B_0	constant magnetic flux density
v	kinematic viscosity
Nu	Nusselt number
Nu_1	Nusselt number on the outer surface of the inner cylinder
Nu_2	Nusselt number on the inner surface of the outer cylinder
P	dimensionless pressure
r	dimensional radial Coordinate
R	dimensionless radial Coordinate
R^*	ratio of radii (R_1/R_2)
R_1	radius of inner cylinder
R_2	radius of outer cylinder
T	dimensional temperature
T_f	reference temperature
$\bar{T}$	average temperature
T_1	temperature of inner cylinder
T_2	temperature of outer cylinder
u	Axial velocity
U_0	mean velocity
V	Dimensionless Velocity
z	axial coordinate

Greek letters

β	degree of asymmetry
θ	dimensionless temperature
θ_b	dimensionless bulk temperature
σ	electrical conductivity of the fluid
μ	dynamic viscosity
ρ	density